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π, the primes and the Lambert$W$ function

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The talk is divided into two parts, the first part will show how to use the bootstrap method to get a formula to calculate the arguments of $\zeta\left(\frac{1}{2}+in\right)$ and a spectacular formula for the n'th zero of the Zeta function using Lambert$W$ function.

The second part will show new formulas for primes like

$$691 = 2^4 \sum_{n=1}^{\infty} \frac{n^{11}}{e^{n\pi}-1} - 2^{16} \sum_{n=1}^{\infty} \frac{n^{11}}{e^{4n\pi}-1}$$

At the same time, the prime 691 is well approximated with the formula

$$691 \approx \frac{2^4 11!}{\pi^{12}}$$

In fact, the prime 691 is given exactly by

$$691 = \frac{2^4 11!}{\pi^{12}} \left(1 + \frac{1}{3^{12}} + \frac{1}{5^{12}} + \frac{1}{7^{12}} + \dots\right)$$

Using the bootstrap method, one can do the same for many primes. This leads to a conjecture about the representation of all the primes using π and a simple function of n. And speaking of primes, I will show a set of formulas that can generate an infinity of primes using a recurrence equation function. If $\{x\}$ is the rounded value of x and $S_0 = 43.804\ldots$, then $S_{n+1} = \left\{S_n^{5/4}\right\}$ will generate an infinity of primes, beginning with

$$113, 367, 1607, 10177, 102217, 1827697, 67201679, 6084503671, \dots$$

Here, the exponent 5/4 can be made as close as we want to 1.